N1 $\xi \sim G(p)$, $p = \frac{1}{2}$

$$\mathbb{P}\{\xi = k\} = p q^{k-1} = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{k-1} = \left(\frac{1}{2}\right)^{k} = \frac{1}{2^k}$$

$E\xi = 2$, $D\xi = 2$,

N2

2) Совместное распредел...

$\xi \backslash \eta$	-1	0	1
-1	1/6	1/12	1/12
1	1/3	1/6	1/6

... Вычислить $cov(\xi\eta, 2\xi$ ...

ξ:

-1	1
$\frac{1}{3}$	$\frac{2}{3}$

η:

-1	0	1
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

$\zeta = \xi\eta$

ζ:

-1	0	1
$\frac{5}{12}$	$\frac{1}{4}$	$\frac{1}{3}$

$$cov(\xi\eta, 2\xi - \eta) = E\xi\eta(2\xi - \eta) -$$

$$- E\xi\eta\, E(2\xi - \eta) = 2E\xi^2\eta - E\xi\eta^2 -$$

$$-2E\xi\eta E\xi + E\xi\eta E\eta =$$

$$= 2E\zeta\xi - E\zeta\eta - 2E\zeta E\xi + E\zeta E\eta$$

$\xi \backslash \zeta$	-1	0	1
-1	1/12	1/12	1/6
1	1/3	1/6	1/6

$\eta \backslash \zeta$	-1	0	1
-1	1/3	0	1/6
0	0	1/4	0
1	1/12	0	1/6

$$2\left(\frac{1}{12}-\frac{1}{6}-\frac{1}{3}+\frac{1}{6}\right)-\left(\frac{1}{3}-\frac{1}{6}-\frac{1}{12}+\frac{1}{6}\right)-$$

$$-2\left(\frac{1}{3}-\frac{5}{12}\right)\left(\frac{2}{3}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{5}{12}\right)\left(\frac{1}{4}-\frac{1}{2}\right)=$$

$$=2\cdot-\frac{1}{4}-\frac{1}{4}+2\cdot\frac{1}{36}+\frac{1}{48}=-\frac{97}{144}$$

№3

$$p_\xi(x)=\begin{cases} c(1-x^2), & x\in[-1,1] \\ 0, & \text{иначе}\end{cases}$$

$$\int_{-1}^{1} c(1-x^2)dx = c\left(\int_{-1}^{1}dx-\int_{-1}^{1}x^2dx\right)=$$

$$=c\left(x\Big|_{-1}^{1}-\frac{x^3}{3}\Big|_{-1}^{1}\right)=\frac{4}{3}c=1 \Rightarrow c=\frac{3}{4}$$

$$F_\xi(x)=\int_{-1}^{x}\frac{3}{4}(1-t^2)dt=\frac{3}{4}\left(t\Big|_{-1}^{x}-\frac{t^3}{3}\Big|_{-1}^{x}\right)=$$

$$=\frac{3}{4}\left(x+1-\frac{x^3}{3}+\frac{1}{3}\right)=-\frac{x^3}{4}+\frac{3}{4}x+1$$

$$F_\xi(x) = \begin{cases} 0, & x < -1 \\ -\frac{x^3}{4} + \frac{3}{4}x + 1, & x \in [-1, 1] \\ 1, & x > 1 \end{cases}$$

$$\mathbb{P}(-0{,}25 < \xi < 0{,}75) = F_\xi(0{,}75) - F_\xi(-0{,}25)\ \textcolor{red}{+1?}$$

$$= 0{,}640625$$

$$\eta = \xi^2 + 1$$

$$F_\eta(t) = \mathbb{P}\{\eta < t\} = \mathbb{P}\{\xi^2 + 1 < t\} = \mathbb{P}\{\xi^2 < t - 1\} =$$

$$= \mathbb{P}\{-\sqrt{t-1} < \xi < \sqrt{t-1}\} =$$

$$= F_\xi(\sqrt{t-1}) - F_\xi(-\sqrt{t-1}) + 1$$

$$p_\eta = F'_\eta = \frac{p_\xi(\sqrt{t-1})}{2\sqrt{t-1}} + \frac{p_\xi(-\sqrt{t-1})}{2\sqrt{t-1}} = \frac{3(2-t)}{4\sqrt{t-1}}$$

$$t \in [1, 2]$$

$$E\eta = E(\xi^2 + 1) = \frac{3}{4}\int_{-1}^{1}(x^2+1)(1-x^2)\,dx =$$

$$= \frac{3}{4}\int_{-1}^{1}(1-x^4)\,dx = \frac{3}{4}\int_{-1}^{1}dx - \frac{3}{4}\int_{-1}^{1}x^4\,dx =$$

$$= \frac{3}{4}x\Big|_{-1}^{1} - \frac{3x^5}{20}\Big|_{-1}^{1} = \frac{3}{4} + \frac{3}{4} - \frac{3}{20} - \frac{3}{20} =$$

$$= \frac{6}{5}$$

$$E\eta = \frac{3}{4}\int_{-1}^{1} (x^2+1)^2 (1-x^2)\,dx = \frac{52}{35}$$

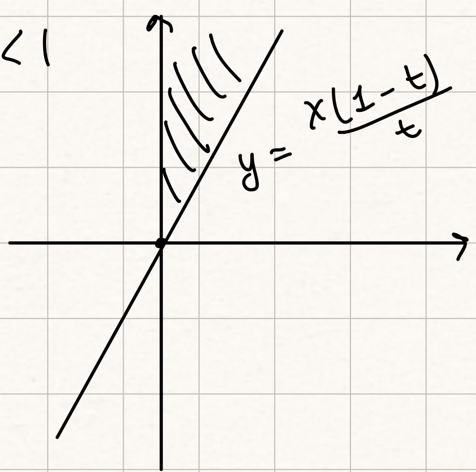
$$D\eta = \frac{52}{35} - \frac{36}{25} = \frac{8}{175}$$

№4 $\quad \xi \sim Exp(\lambda), \ \eta \sim Exp(\lambda)$

$$\zeta = \frac{\xi}{\xi+\eta}$$

$$F_\zeta(t) = \mathbb{P}\left\{\frac{\xi}{\xi+\eta} < t\right\} = \iint\limits_{\frac{x}{x+y} < t} \lambda^2 e^{-\lambda(x+y)}\,dx\,dy =$$

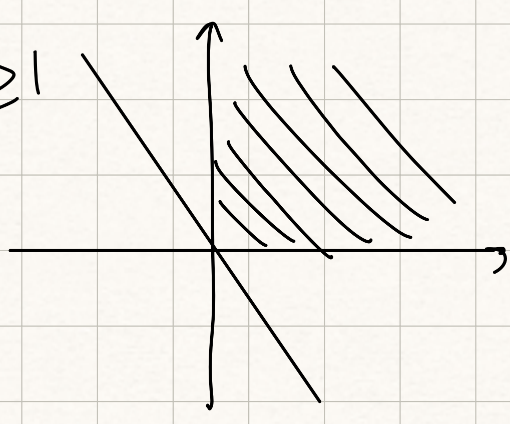
$t < 1$



1) $$\int_0^{+\infty} dy \int_0^{\frac{ty}{1-t}} \lambda^2 e^{-\lambda x} e^{-\lambda y}\,dx =$$

$$= \int_0^{+\infty} -\lambda e^{-\frac{\lambda y}{1-t}}\,dy =$$

$t \geq 1$



$$= (1-t)e^{-\frac{\lambda y}{1-t}}\Big|_0^{+\infty} =$$

$$= t - 1$$

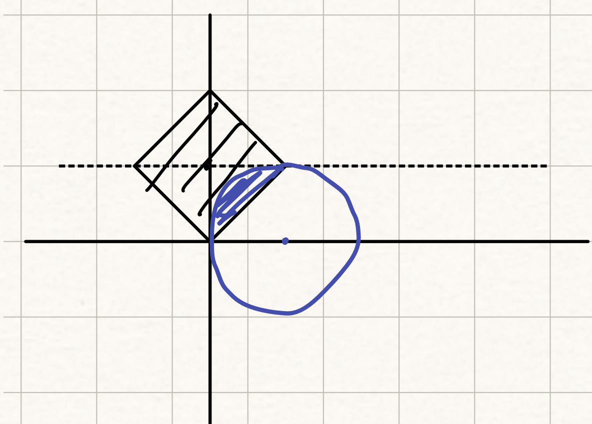
2) $t \geq 1$ $\quad \int_0^{+\infty} dx \int_0^{+\infty} \lambda^2 e^{-\lambda x} e^{-\lambda y} dy =$

$$= \int_0^{+\infty} \lambda e^{-\lambda x} dx = - e^{-\lambda x} \Big|_0^{+\infty} = 1$$

$$P_\xi(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{иначе} \end{cases}$$ – равн. распр-е

$E\xi = \frac{1}{2} \quad D\xi = \frac{1}{12}$

№5 $\quad p(x,y) = c \;, \quad |x| + |y-1| \leq 1$



$$\iint\limits_{|x|+|y-1|\leq 1} c \, dx dy = cS = 2c = 1$$

$$c = \frac{1}{2}$$

$$\mathbb{P}\{(\xi-1)^2 + \eta^2 \leq 1\} = \iint\limits_{\substack{|x|+|y-1|\leq 1 \\ (x^2-1)+y^2\leq 1}} \frac{1}{2} dx dy =$$

$$= \int_{\pi/4}^{\pi/2} d\varphi \int_{0}^{2\cos\varphi} \frac{z}{2}\, dz = \int_{\pi/4}^{\pi/2} \cos^2\varphi \, d\varphi =$$

$$= \frac{1}{2} \int_{\pi/4}^{\pi/2} d\varphi + \frac{1}{4} \int_{\pi/4}^{\pi/2} \cos 2\varphi \, d(2\varphi) =$$

$$= \frac{1}{2} \varphi \Big|_{\pi/4}^{\pi/2} + \frac{1}{4} \sin 2\varphi \Big|_{\pi/4}^{\pi/2} = \frac{\pi}{8} - \frac{1}{4}$$

$$p_\xi = \begin{cases} \int_{-x}^{2+x} c\, dy, & x \in [-1, 0] \\ \int_{x}^{2-x} c\, dy, & x \in [0, 1] \\ 0, & \text{иначе} \end{cases} =$$

$$= \begin{cases} 1 + x, & x \in [-1, 0] \\ 1 - x, & x \in [0, 1] \\ 0, & \text{иначе} \end{cases}$$